This spring, I presented my senior colloquium on a topological game called the Banach-Mazur Game. This game was formulated by Stanislaw Mazur in 1935. It is the earliest example of an infinite game with perfect information. The game proceeds as followed. Consider some arbitrary closed interval I_0 in $\mathbb{R}$. There are two players in this game: Alice and Bob. Alice is dealt an arbitrary subset A of I_0. The complementary set, which we denote B, is dealt to Bob. The rules of the game are as follows: Alice chooses an arbitrarily closed interval $I_1 \subset I_0$; Then Bob selects a closed interval $I_2 \subset I_1$. After this, Alice then chooses a closed interval $I_3 \subset I_2$ and so on. This looks like:

$$I_n \subset ...I_4 \subset I_3 \subset I_2 \subset I_1$$

Together, Alice and Bob determine a sequence of nested, closed intervals I_n with Alice choosing those of an odd index and Bob choosing those of an even index. We say that Alice wins if the set $\cap I_n$ has at least a single point in common with A. If this is not the case, then we say that Bob wins. Put more formally, we write that Alice wins if:

$$\cap_{i=1}^{\infty} I_n \cap A \neq 0$$

Similarly, Bob wins if:

$$\cap_{i=1}^{\infty} I_i \subset B$$

The question I was interested in was: when does there exist a winning strategy for either Alice or Bob?

The answers to the above question can be found in the theorem.

Banach-Mazur Game In the Banach-Mazur Game outlined above, there exists a winning strategy for Bob if and only if the set A is meager.

Intuitively, we get the sense that Alice would win if her set is, in some sense, large relative to Bob's. The theorem reveals that Alice's set is"large" if the set is not meager - a topological property which means that the set A can be written as countably many nowhere dense subsets of $\mathbb{R}$. Banach proved the theorem (unpublished) and Oxtoby published a proof in 1957. Moreover, by the symmetric nature of the game, one can show that there exists a winning strategy for Bob if and only for some $I \subset I_0$ $B \cap I$ is meager. Oxtoby also showed that, accepting the validity of the Axiom of Choice, there exist variations of the Banach-Mazur game such that $\forall I \subset I_0$ neither $A \cap I$ nor $B \cap I$ is meager. Thus neither player possesses a winning strategy. This result is paradoxical, because the game is infinite and there is no way in its construction to allow for a tie.

What interested me as I was researching this problem was the history of the Banach-Mazur Game. As mentioned above, the game was posed by Stanislaw Mazur a mathematician from Lviv (*Lwów*), a medium-sized city near the Polish-Ukranian border. In the first half of the twentieth century Lviv housed a group of scholars known as the Lviv School of Mathematics.This group included mathematical luminaries such as Stefan Banach Stanislaw Ulam,and Mark Kac. So Lviv had become a center of mathematical activity in Europe with an active seminar series. These seminars were held on Saturday, usually in the late afternoon or the evening, and following the talks the researchers would go out to the cafe, known as the Scottish Cafe, to discuss what they had learned, pose open problems, and tease out some of the interesting mathematical implications of the research.

However, the group would write all of their work on the tables in the cafe which would be wiped clean after every session. Banach thought that it would be advantageous to keep some sort of repository for the open questions and problems that they were working on. Thus he bought a thickly bound notebook which was stored behind the counter of the restaurant. Now, when the researchers would come into the to work, the waiter would bring them the notebook along with their coffee or tea. This book became known as the Scottish Book, An electronic copy of this book can be accessed for Williams students through Francis, and I encourage everyone to take a look, as the problems themselves are very neat and are accompanied by some interesting commentary. One quote from Daniel Mauldin found in the preface to the second edition of the Scottish Book (Springer, 2015) particularly resonated with me.

> It (the Scottish Book) represents the best of cafe mathematics, an informal and free wheeling style of mathematical conversation and interaction that seems almost lost today. (Mauldin, v)

What struck me about this quote was is that "cafe Mathematics", the spirit of scholastic excellence through collaboration and bonhomie, is certainly not lost at the Williams College Department of Mathematics. I will be graduating from Williams in a few weeks, and some of my fondest memories have been the afternoons that I have spent in Bronfman library working with my friends on Algebra or Differential Equations problem sets, poking in to offices when I get hung up on problems, and asking for help from the professors who circle the room. The spirit of the Scottish Cafe lives on in Math/Stats at Williams, it is what makes the department at Williams so unique and what has made my tenure as a math major enjoyable and enriching. I look forward to visiting as an alumnus, and seeing how the Math community at Williams continues to foster a community of students who are excited about math.