Abstracts of Papers
of
Thomas Garrity

1. On Ample Vector Bundles and Negative Curvature, thesis, Brown University, 1986.

   A good reference to this work is in S. Lang's *Introduction to Complex Hyperbolic Spaces*, V.4 "Garrity's Theorem."

2. On Computing the Intersection of a Pair of Algebraic Surfaces (with J. Warren), *Computer Aided Geometric Design*, (1989), Vol. 6, pp. 137-154.

   The problem of finding the intersection of a pair of surfaces arises in a wide range of applications in geometric modeling. One difficulty in computing the intersection is that there has been no data structure suitable for representing arbitrary algebraic space curves. In this work, we describe such a data structure for the intersection of surfaces. This data structure represents an irreducible space curve as an irreducible algebraic plane curve plus a birational map between the plane curve and the space curve. Furthermore, we give an algorithm for constructing such a representation for the space curve formed by two intersecting algebraic surfaces. Finally, we show how this data structure may be used in algorithms to answer important questions about space curves.

3. Factoring Rational Polynomials Over the Complexes (with C. Bajaj, J. Canny and J. Warren), *Proc. 1989 Intl. Symp. Symbolic Algebraic Comput.*, ACM (1989), pp. 81- 90.

   An earlier version of (5), in which we determine the number and degree of each irreducible factor of a rational polynomial in NC but only approximate the factors in polynomial time, which is slower than NC.

4. Geometric Continuity (with J. Warren), *Computer Aided Geometric Design*, (1991), Vol. 8, pp. 51-66.

   We state and motivate a general definition of geometric continuity. In the case of analytic sets, we drive a general algebraic characterization of geometric continuity. We then use this characterization to show that the following representation-related notions are equivalent:

(a) Derivative continuity for explicitly-defined surfaces.

(b) Rescaling continuity for implicitly-defined surfaces.

(c) Reparameterization continuity for parametrically-defined surfaces.

(d) Intersection multiplicity for algebraic surfaces.

In the implicit case, a new, equational characterization of geometric continuity is given and used to derive implicit curves and surfaces that meet with a given order of continuity.

5. Factoring Rational Polynomials Over the Complex Numbers (with C. Bajaj, J. Canny and J. Warren), *SIAM J. of Computation*, (1993), v. 22, n. 2, pp. 318-342.

NC algorithms are given for determining the number and degrees of the factors, irreducible over the complex numbers C, of a multivariate polynomial with rational coefficients and for approximating each irreducible factor. NC is the class of functions computable by logspace-uniform Boolean circuits of polynomial size and polylogarithmic depth. The measure of size of the input polynomial are its degree d, coefficient length c, number of variables n. If n is fixed, we give a deterministic NC algorithm. If the number of variables is now fixed, we give a random (Monte-Carlo) NC algorithm in these input measures to find the number and degree of each irreducible factor.

After reducing to the two variable, square-free case, we use the classical algebraic geometry fact that the absolute irreducible factors of $(P(z_1, z_2) = 0)$ correspond to the connected components of the real surface (or complex curve) $(P(z_1, z_2) = 0)$ minus its singular points. To find the number of connected components of the surface $P = 0$, project the surface to the z_2- plane. The singular points of $(P(z_1, z_2) = 0)$ will lie over the projection's critical values, yielding that the inverse image of a grid isolating the critical values in the z_2 plane will lift to a one-dimensional real curve on the surface $(P(z_1, z_2) = 0)$, whose number of connected components will be precisely the number of connected components of the surface. The adjacency matrix of this graph is constructed by treating symbolically, via Sturm sequences, the zeros of the various polynomials defining the maps. Once we have the number of irreducible factors and their degrees, we can approximate the actual factors by using the recent result of Neff [22] on finding zeros of one variable polynomials in NC.

6. Invariants of Vector-Valued Bilinear and Sesquilinear Forms (with R. Mizner), *Linear Algebra and its Applications* , (1995), v. 218, pp. 225-237.

   First steps in the algebraic invariant theory of vector-valued bilinear and sesquilinear forms are made. In particular, explicit formulas for generators of all invariant rational functions of such forms are derived. These formulas, and certain analogues, have applications to the geometry of Riemannian submanifolds, distributions, and CR structures.

7. The Equivalence Problem for Higher Codimensional CR Structures (with R. Mizner), *Pacific J. of Math.*, (1997), v. 177, no. 2, pp. 211-235.

   The equivalence problem for CR structures can be viewed as a special case of the equivalence problem for G-structures. This paper uses Cartan's methods (in modernized form) to show that a CR manifold of codimension 3 or greater with suitably generic Levi form admits a canonical connection on a reduced structure bundle whose group is isomorphic to the multiplicative group of complex numbers. As corollaries, it follows that the CR manifold admits a canonical affine connection, and consequently that the automorphisms of the CR manifold constitute a Lie group.

   The most difficult technical step is to construct a smooth moduli space for generic vector-valued hermitian forms, which is tied to the CR manifold via the Levi map. The techniques used to construct this space are drawn from the classical invariant theory of complex projective hypersurfaces.

8. Vector-Valued Forms and CR Geometry (with R.Mizner), **Advanced Studies in Pure Mathematics**, (1997), vol. 25: CR-Geometry and Overdetermined Systems, pp. 110-121.

   Vector-valued forms arise in the study of various higher codimensional geometries. This note gives an overview of how the invariant theory of the Levi form (a vector- valued form) can be used to understand higher codimensional CR-structures.

9. Intersection Forms and the Adjunction Formula for Four-manifolds via CR Geometry(with M. Chkhenkeli), (1999), preprint available at front.math.ucdavis.edu/math.DG/9904007.

   This is primarily an expository note showing that earlier work of Lai on CR geometry provides a clean interpretation, in terms of a Gauss map, for an adjunction formula for embedded surfaces in an almost complex four manifold. We will see that if F is a surface with genus g in an almost complex four-manifold M, then

   $$2 - 2g + F \cdot F - i^* c_1(M) - 2F \cdot C = 0,$$

   where C is a two-cycle on M pulled back from the cycle of two planes with complex structure in a Grassmannian $Gr(2, C^N)$ via a Gauss map and where $i^* c_1(M)$ is the restriction of the first Chern class of M to F. The key new term of interest is $F \cdot C$, which will capture the points of F whose tangent planes inherit a complex structure from the almost complex structure of the ambient manifold M. These complex jump points then determine the genus of smooth representatives of a homology class in $H_2(M, Z)$. Further, via polarization, we can use this formula to determine the intersection form on M from knowing the nature of the complex jump points of M's surfaces.

10. On periodic sequences for algebraic numbers, *J. of Number Theory*, (2001), Vol. 88, pp. 86-103.

    For each positive integer $n \geq 2$, a new approach to expressing real numbers as sequences of nonnegative integers is given. The $n = 2$ case is equivalent to the standard continued fraction algorithm. For $n = 3$, it reduces to a new iteration of the triangle. Cubic irrationals that are roots of $x^3 + kx^2 + x - 1$ are shown to be precisely those numbers with purely periodic expansions of period length one. For general positive integers n, it reduces to a new iteration of an n dimensional simplex.

11. Global Structures on CR-Manifolds via Nash Blow-ups, *Michigan J. of Mathematics*, (2001), Vol. 48, pp. 281-294.

    A generic compact real codimension two submanifold X of C^{n+2} will have a CR structure at all but a finite number of points (failing at the complex jump points J). The main theorem of this paper gives a method of extending the CR structure on the non-jump points $X - J$

to the jump points. We examine a Gauss map from $X - J$ to an appropriate flag manifold F and take the closure of the graph of this map in $X \times F$. This is a version of a Nash blow-up. We give a clean criterion for when this closure is a smooth manifold and see that the local differential properties at the points $X - J$ can now be naturally extended to this new smooth manifold, allowing global techniques from differential geometry to be applied to compact CR manifolds. As an example, we find topological obstructions for the manifold to be Levi nondegenerate.

12. *All the Mathematics You Missed [But Need to Know for Graduate School]* ,Cambridge University Press, January 2002, 347 pages plus xvii.

    A broad survey of what is needed for graduate work in mathematics.

13. Texts in Algebraic and Differential Geometry, chapter in **Using the Mathematics Literature**, edited by Kristine Fowler, Marcel Dekker, 2004.

    A survey of basic texts in both Algebraic Geometry and Differential Geometry.

14. On Relations of Invariants for Vector-Valued Forms (with Z. Grossman), *Electronic Journal of Linear Algebra*, (2004), Vol. 11, pp. 24-40.

    A method for constructing the syzygies of vector-valued bilinear forms is given. Explicit formulas for the generators of the relations among the invariant rational functions are listed. These formulas have applications in the geometry of Riemannian submanifolds and in CR geometry.

15. A Two-Dimensional Minkowski ?(x) Function (with O. Beaver), *Journal of Number Theory*, (2004), Vol. 107 no. 1, 105–134.

    A one-to-one continuous function from a triangle to itself is defined that has both interesting number theoretic and analytic properties. This function is shown to be a natural generalization of the classical Minkowski ?(x) function. It is shown there exists a natural class of pairs of cubic irrational numbers in the same cubic number field that are mapped to pairs of rational numbers, in analog to ?(x) mapping quadratic irrationals on the unit interval to rational numbers on the unit interval. It is also shown that this new function satisfies an analog to the fact that ?(x), while increasing and continuous, has derivative zero almost everywhere.

16. A Dual Approach to Triangle Sequences: A Multidimensional Continued Fraction Algorithm (with S. Assaf, L. Chen, T. Cheslack-Postava, B. Cooper, A. Diesl, M. Lepinski and A. Schuyler), *Integers*, (2005), Vol. 5 no. 1, A8, 39 pp.

    A dual approach to defining the triangle sequence (a type of multidimensional continued fraction algorithm, initially developed in 10) for a pair of real numbers is presented, providing a new, clean geometric interpretation of the triangle sequence. We give a new criterion for when a triangle sequence uniquely describes a pair of numbers and give the first explicit examples of triangle sequences that do not uniquely describe a pair of reals. Finally, this dual approach yields that the triangle sequence is topologically strongly mixing, meaning in particular that it is topologically ergodic.

17. Review of John Adam's **Mathematics in Nature: Modeling Patterns in the Natural World**, *Mathematical Intelligencer*, (2005); Vol. 27 (2), p. 81.

18. THE GREAT Pi/e DEBATE (DVD), (with C. Adams and E. Burger), MAA, 2007.

    A light hearted debate suitable for any audience from middle school on up.

19. Teaching Tips, with Frank Morgan, www.ams.org/profession, (as of Sept. 2008)

    Almost all of us do some teaching. All of us could do better. If doing better required a lot of time or a drastic overhaul of our basic personalities, we probably wouldn't bother. Here is a list of easy ways to be a better teacher. In the epilogue we talk a bit about teaching styles.

20. UNITED STATES OF MATH PRESIDENTIAL DEBATE (DVD), (with C. Adams and E. Burger), MAA, 2009.

    A debate between the figure 8 knot and the Euclidean Algorithm as to who should be the next president of the United States of Mathematics. Suitable for any audience from high school on up.

21. On a Thermodynamic Classification of Real Numbers, *Journal of Number Theory*, 2010, Vol. 130, Issue 7, pp. 1537-1559.

    A new classification scheme for real numbers is given, motivated by ideas from statistical mechanics in general and work of Knauf and Fiala and Kleban in particular. Critical for this classification of a real number will be the Diophantine properties of its continued fraction expansion.

22. Using Mathematical Maturity to Shape our Teaching, our Careers and our Departments, *Notices of the American Mathematical Society*, December 2011, pp.1592-1593. (An expanded version is at http://arxiv.org/pdf/1410.4255.pdf)

    A short article on how the idea of mathematical maturity can be used to influence all parts of our professional life.

23. DERIVATIVE VERSUS INTEGRAL; THE FINAL SMACKDOWN (DVD), (with C. Adams and A. Falk), MAA, 2012.

    A debate on which is more important: the derivative or the integer. Suitable for any audience from high school on up.

24. *Algebraic Geometry: A Problem Solving Approach* (with R. Belshoff, L. Boos, R. Brown, J. Douilhet, C. Lienert, D. Murphy, J. Navarra-Madsen, P. Poitevin, S. Robinson, B. Synder, C. Werner), American Mathematical Society, Student Mathematical Library, Vol. 66, 2013.

    Algebraic Geometry has been at the center of much of mathematics for hundreds of years. It is not an easy field to break into, despite its humble beginnings in the study of circles, ellipses, hyperbolas and parabolas. This text consists of a series of exercises, starting with conics and ending with sheaves and cohomology. The first chapter on conics is appropriate for first year college students (and many high school students). Chapter two leads the reader to an understanding of the basics of cubic curves, while chapter three provides an introduction to higher degree curves. Both chapters are appropriate for people who have taken multivariable calculus and linear algebra. Chapters four and five provide an introduction to geometric objects of higher dimension than curves. Here abstract algebra plays a critical role, meaning in part that these chapters are for people who have taken a first course in abstract algebra. The last chapter is on sheaves and cohomology, providing a hint of current work in algebraic geometry.

25. A Multidimensional Continued Fraction Generalization of Stern's Diatomic Sequence, *Journal of Integer Sequences* **16** (2013), Article 13.7.7, available at
https://cs.uwaterloo.ca/journals/JIS/VOL16/Garrity/garrity4.html

    Continued fractions are linked to Stern's diatomic sequence

    $$0, 1, 1, 2, 1, 3, 2, 3, 1, 4, \dots$$

    (given by the recursion relation $\alpha_{2n} = \alpha_n$ and $\alpha_{2n+1} = \alpha_n + \alpha_{n+1}$, where $\alpha_0 = 0$ and $\alpha_1 = 1$), which has long been known. Using a particular multi-dimensional continued fraction algorithm (the Farey algorithm), we will generalize the diatomic sequence to a sequence of numbers that quite naturally should be called Stern's triatomic sequence (or a two-dimensional Pascal's sequence with memory). As continued fractions and the diatomic sequence can be thought of as coming from a systematic subdivision of the unit interval, this new triatomic sequence will arise by a systematic subdivision of a triangle. We will discuss some of the algebraic properties for the triatomic sequence.

26. A thermodynamic classification of pairs of real numbers via the Triangle Multi-dimensional continued fraction, available at
http://arxiv.org/pdf/1205.5663.pdf

    A new classification scheme for pairs of real numbers is given, generalizing earlier work of the author that used continued fraction, which in turn was motivated by ideas from statistical mechanics in general and work of Knauf and Fiala and Kleban in particular. Critical for this classification are the number theoretic and geometric properties of the triangle map, a type of multi-dimensional continued fraction.

27. A Generalized Family of Multidimensional Continued Fractions: TRIP Maps (with Krishna Dasaratha, Laure Flapan, Chansoo Lee, Cornelia Mihaila, Nicholas Neumann-Chun, Sarah Peluse Matt Stoffregen), *International Journal of Number Theory*, (2014), vol. 10, No. 08 , pp. 2151-2186.

    Most well-known multidimensional continued fractions, including the Mönkemeyer map and the triangle map, are generated by repeatedly subdividing triangles. This paper constructs a family of multidimensional continued fractions by permuting the vertices of these triangles before and after each subdivision. We obtain an even larger class of multidimensional continued fractions by composing the maps in the family. These include the algorithms of Brun, Parry-Daniels and Güting. We give criteria for when multidimensional continued fractions associate sequences to unique points, which allows us to determine when periodicity of the corresponding multidimensional continued fraction corresponds to pairs of real numbers being cubic irrationals in the same number field.

28. Cubic Irrationals and Periodicity via a Family of Multi-dimensional Continued Fraction Algorithms (with Krishna Dasaratha, Laure Flapan, Chansoo Lee, Cornelia Mihaila, Nicholas Neumann-Chun, Sarah Peluse Matt Stoffregen), Monatshefte für Mathematik, (2014), Vol. 174, Number 4, pp. 549-566.

    We construct a countable family of multi-dimensional continued fraction algorithms, built out of five specific multidimensional continued fractions, and show a real number is a cubic irrational precisely when its multidimensional continued fraction expansion with respect to at least one element of the countable family is eventually periodic. We interpret this result as the construction of a matrix with entries of non-negative integers such that at least one of the rows is eventually periodic if and only if the chosen real is a cubic irrational. This result is built on a careful technical analysis of certain units in cubic number fields and our family of multi-dimensional continued fractions.

29. Review of *Spherical tube hypersurfaces,*by Alexander Isaev., *Bulletin of the American Mathematical Society* (2014), Vol. 51, Number 4, pp. 675-685.

    Using this text as a springboard, we discuss why CR geometries are natural objects of study in several complex variables, partial differential equations and differential geometry.

30. *Electricity and Magnetism for Mathematicians: A Guided Path from Maxwell's Equations to Yang-Mills*, Cambridge University Press, 2015.

    This text is an introduction to some of the mathematical wonders of Maxwell's equations. These equations led to the prediction of radio waves, the realization that light is an electro-magnetic wave, and the discovery of the special theory of relativity. In fact, almost all current descriptions of the fundamental laws of the universe can be viewed as deep generalizations of Maxwell's equations. Even more surprising is that these equations and their generalizations have led to some of the most important mathematical discoveries of the past thirty years. It seems that the mathematics behind Maxwell's equations is endless.

The goal of this book is to explain to mathematicians the underlying physics behind electricity and magnetism and to show their connections to mathematics. Starting with Maxwell's equations, the reader is led to such topics as the special theory of relativity, differential forms, quantum mechanics, manifold, tangent bundles, connections and curvature.

31. Review of *Differential Forms: Theory and Practice* by Steven Weintraub, *American Mathematical Monthly*, vol. 123, No. 4 (April 2016), pp. 407-412.

    In the context of this text we discuss why, and to a lesser extent how, to teach differential forms, with a emphasis on teaching at the undergraduate level.

32. Pedersen & Tom videos. with L. Pedersen, available at www.youtube.com/channel/UCNcvTX3a_jyT4U_G3C-Y0VQ

    A series of light hearted humorous sketches explaining some basic mathematics.

33. A Framework for Multidimensional Continued Fractions, currently available at www.irif.fr/ dyna3s/Comp

    This is not a paper. It is an attempt to fix, for multidimensional continued fractions, notation that many of us can then use. The first section is for $\mathbb{R}^n$. Section two gives the eight triangle partition map analogs of the classical Gauss map, the $\mathbb{R}^2$ case. I have not seen them before but would not be at all surprised if they are already known. If anyone has seen one of the seven "new" maps before, please let me know. The third section is fixing notation for the $\mathbb{R}^3$ case. The original triangle map is worked out, as well as showing that the Cassaigne algorithm is a TRIP map. The final section is about what a common framework might look like. These notes would not be appropriate for someone who did not already know something about multi-dimensional continued fractions. Comments and corrections welcome.

34. Stern Sequences for a Family of Multidimensional Continued Fractions: TRIP-Stern Sequences (with Ilya Amburg, Krishna Dasaratha, Laure Flapan, Chansoo Lee, Cornelia Mihaila, Nicholas Neumann-Chun, Sarah Peluse Matt Stoffregen), *Journal of Integer Sequences* **20** (2017), Article 17.1.7, available at
https://cs.uwaterloo.ca/journals/JIS/VOL20/Garrity/garrity6.pdf

    The Stern diatomic sequence is closely linked to continued fractions via the Gauss map on the unit interval, which in turn can be understood via systematic subdivisions of the unit interval. Higher dimensional analogues of continued fractions, called multidimensional continued fractions, can be produced through various subdivisions of a triangle. We define triangle partition-Stern sequences (TRIP-Stern sequences for short), higher-dimensional generalizations of the Stern diatomic sequence, from the methods of subdividing the triangle presented in paper 28. We then explore several combinatorial results about TRIP-Stern sequences, which may be used to give rise to certain well-known sequences. We finish by generalizing TRIP-Stern sequences and presenting analogous results for these generalizations.

35. Generalizing the Minkowski Question Mark Function to a Family of Multidimensional Continued Fractions, with Peter McDonald, *International Journal of Number Theory*, Vol. 14, No. 09, pp. 2473-2516 (2018)

    The Minkowski question mark function, maping the unit interval to itself, is a continuous, strictly increasing, one-to-one and onto function that has derivative zero almost everywhere. Key to these facts are the basic properties of continued fractions. Thus the question mark function is a naturally occurring number theoretic singular function. This paper generalizes the question mark function to the 216 triangle partition (TRIP) maps. These are multidimensional continued fractions which generate a family of almost all known multidimensional continued fractions. We show for each TRIP map that there is a natural candidate for its analog of the Minkowski question mark function. We then show that the analog is singular for 96 of the TRIP maps and show that 60 more are singular under an assumption of ergodicity.

36. Functional Analysis behind a Family of Multidimensional Continued Fractions: Triangle Partition Maps I, with Ilya Amburg, *Publicationes*

Mathematicae Debrecen, Vol. 98 (2021), no. 1-2, 43-63.

Triangle partition maps form a family that includes many, if not most, well-known multidimensional continued fraction algorithms. This paper begins the exploration of the functional analysis behind the transfer operator of each of these maps. We show that triangle partition maps give rise to two classes of transfer operators and present theorems regarding the origin of these classes; we also present related theorems on the form of transfer operators arising from compositions of triangle partition maps. In the next paper, Part II, we will find eigenfunctions of eigenvalue 1 for transfer operators associated with select triangle partition maps on specified Banach spaces and then proceed to prove that the transfer operators, viewed as acting on one-dimensional families of Hilbert spaces, associated with select triangle partition maps are nuclear of trace class zero. We will finish in part II by deriving Gauss-Kuzmin distributions associated with select triangle partition maps.

37. Functional Analysis behind a Family of Multidimensional Continued Fractions: Triangle Partition Maps II, with Ilya Amburg, *Publicationes Mathematicae Debrecen*, Vol. n 98 (2021), no. 3-4, 259-276.

    This paper is a direct continuation of ”Functional analysis behind a Family of Multidimensional Continued Fractions: Part I,” in which we started the exploration of the functional analysis behind the transfer operators for triangle partition maps, a family that includes many, if not most, well-known multidimensional continued fraction algorithms. This allows us now to find eigenfunctions of eigenvalue 1 for transfer operators associated with select triangle partition maps on specified Banach spaces. We proceed to prove that the transfer operators, viewed as acting on one-dimensional families of Hilbert spaces, associated with select triangle partition maps are nuclear of trace class zero. We finish by deriving Gauss-Kuzmin distributions associated with select triangle partition maps.

38. *All the Math You Missed: But Need to Know for Graduate School*, Cambridge University Press, second edition (four new chapters), July 2021, 416 pages.

    Second edition of 12, with four new chapters, on Elementary Number Theory, Algebraic Number Theory, Analytic Number Theory and

Category Theory.

39. On Gauss-Kuzmin Statistics and the Transfer Operator for a Multidimensional Continued Fraction Algorithm: the Triangle Map, to appear in *Publicationes Mathematicae Debrecen*, available at http://arxiv.org/pdf/1509.01840v1.pdf

    The Gauss-Kuzmin statistics for the triangle map (a type of multidimensional continued fraction algorithm) are derived by examining the leading eigenfunction of the triangle map's transfer operator. The technical difficulty is finding the appropriate Banach space of functions. We also show that, by thinking of the triangle map's transfer operator as acting on a one-dimensional family of Hilbert spaces, the transfer can be thought of as a family of nuclear operators of trace class zero.

40. On Partition Numbers and Continued Fraction Type Algorithms, with Claudio Bonanno Alessio Del Vigna and Stefano Isola, submitted, available at https://arxiv.org/abs/2109.08962

    We show that the additive-slow-Farey version of the traditional continued fractions algorithm has a natural interpretation as a method for producing integer partitions of a positive number n into two smaller numbers, with multiplicity. We provide a complete description of how such integer partitions occur and of the conjugation for the corresponding Young shapes via the dynamics of the classical Farey tree. We use the dynamics of the Farey map to get a new formula for p(2,n), the number of ways for partitioning n into two smaller positive integers, with multiplicity. We then do the analogue using the additive-slow-Farey version of the Triangle map (a type of multi-dimensional continued fraction algorithm), giving us a method for producing integer partitions of a positive number n into three smaller numbers, with multiplicity. However different aspects of this generalisations remain unclear.

41. On the linear complexity of triangle partition maps, with Daniel Jordan Alvarez, Amy Bradford, Ding Ding Dong, Konnor Herbst, Ariel Koltun-Fromm, Brian Mintz, Vaughn Osterman and Mary Stelow, in preparation.

    Tentative: Linear complexity bounds are found for all S-adic words stemming from triangle partition maps. In particular, we show that the

Cassaigne algorithm, in a certain well-defined sense, is the only such algorithm with linear complexity $2n$ and that the linear complexity of the triangle map is bounded by $3n$.

42. On Hidden Continued Fractions in Some Multidimensional Continued Fraction Algorithms, with Daniel Jordan Alvarez, Amy Bradford, Ding Ding Dong, Konnor Herbst, Ariel Koltun-Fromm, Brian Mintz, Vaughn Osterman and Mary Stelow, in preparation.